A Quasi-Play about an Unlucky Day

by John M. Boyer, PhD, SFSPE

Characters: I, SomeOne, SomeTwo, SomeThree, NoBody, and You

Sitting around the dinner table one night on a Friday the 13th, SomeOne, who had had an unlucky day, asked, "How many Fridays the 13th can you have in a year, anyway?!?" After a number of milliseconds divisible by 13, I realized that discussing the answer would make for a good example of upper bounds.

So, I started off provocatively by saying, "Well, although this isn't a tight upper bound, you know that it can't be greater than 53."

That turned out to be slightly more fun than I thought, as SomeTwo interjected, "You mean 52, right?"

"No," I replied.

"Oh, you're including leap year!" exclaimed SomeThree.

"Nope, ignore leap year," I said.

SomeThree's brow furrowed as SomeTwo enquired, "Okay, then why 53?"

If you're still reading this, you're a bona fide propeller-head, so you already know two things.

First, a non-leap year has 52 weeks and 1 day; so, if the year did start on a Friday, then the year could include 53 Fridays. Second, you know that this upper bound is ridiculously inaccurate because you can't have a Friday the *13th* on January *1st*—and SomeThree made a good recovery by pointing this out!

"Yes," I agreed, "as a feature, Friday-ness is just not enough. It also has to be the 13th day of a month; and, therefore, 12 is a tighter upper bound than 53."

"Oh, goody," grumped SomeOne, "so this year can be as bad as today 11 more times. Great."

At this point, SomeTwo applied the earlier lesson learned, adding, "Yes, but that's not a *tight* upper bound, either, because there's no way for the 13th days of all months to be on Fridays."

"I like *that* news, but why not?" asked SomeOne.

Catching on, SomeThree enthused, "Oh, because the numbers of the days in each month are all wrong!"

"Except February in a non-leap year," I nudged.

Much finger-counting began, as the nudge got SomeTwo to rally everyone around to calculate, starting with a hypothetical Friday the 13th in February.

SomeThree extolled, "If we assume it's not a leap year, we get a Friday the 13th in March, too; and, surely, there's going to be another one later in the year!" The finger-counting eventually revealed a third Friday the 13th in November of that hypothetical year.

Spirits slightly buoyed, SomeOne declared, "Cool, that's a good upper bound. A year can't get any worse than three days like today."

"*Wellll*," I gently countered, "the good news is that there's only one more Friday the 13th *this* year; but what we just found is that a year could have at *least* three Fridays the 13th in it. To be an upper bound, you'd have to show how many are the *most* Fridays the 13th. So, can you say that there *can't* be four of them in a year?"

Much more finger-counting began, led by SomeTwo, with SomeThree chiming in to report any errors, and SomeOne crossing fingers through more recounts than a twenty-first-century election. It was a lot of work, which had to be done cranially, because NoBody took the pen and paper.

Finally, by the method of exhaustion on possible starting months for the first Friday the 13th, SomeTwo and SomeThree concluded that three was the tight upper bound. I'm telling you, there was so much counting and waggling of fingers that I began to hope we might next start quoting successive primes to each other—but no such luck. It was, after all, Friday the 13th. *Well*, I thought, *at least 13 is prime.*

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Epilogue

The year 2023 has two Fridays the 13th in it: one in January and one in October.

Is that the lower bound? How few Fridays the 13th can a non-leap year have in it? Can there ever be *no* Fridays the 13th in a non-leap year?

For non-leap years that have at least one Friday the 13th, which month is the *latest* in which the first Friday the 13th can occur?

Does being a leap year change the upper bound, the lower bound, or the latest month in which a Friday the 13th could occur?

Let NoBody take the pen and paper as You commence cranial considerations.

(Answers can be found in the *Solutions* section at the back of this issue.) Ω

A Quasi-Play about an Unlucky Day: Answers

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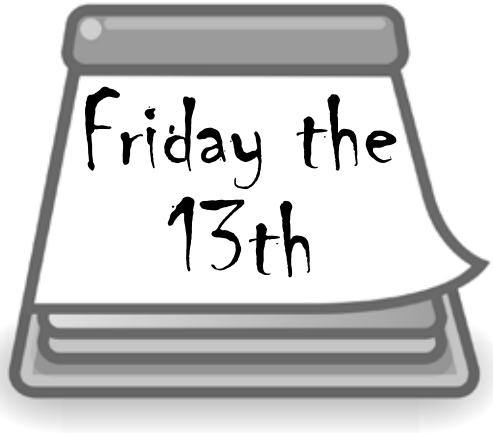
In a non-leap year, 3 was reported to be the upper bound on the number of Fridays the 13th. Every non-leap year has at least 1 Friday the 13th, and the minimum of 1 occurs when Friday the 13th occurs in May, June, or August. However, the year in which one's luck may hold out the longest has not 1 but 2 Fridays the 13th, in September and December.

In a leap year, the upper bound does change, down to 2. The lower bound is 1, and the minimum of 1 occurs when Friday the 13th occurs in May, June, or October, the last of which is, of course, also the longest one's luck will hold out in a leap year. Ω



"Friday the 13th
is still better than
Monday the whatever."
—Unknown