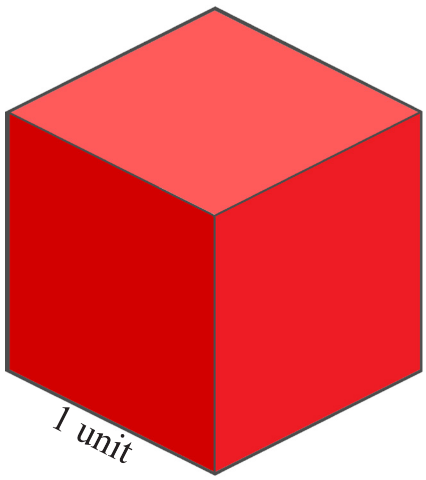
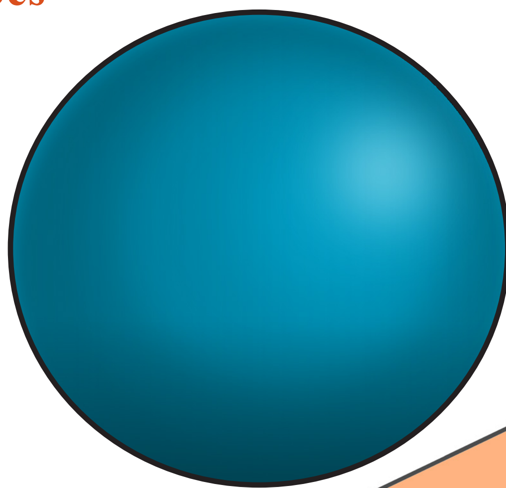
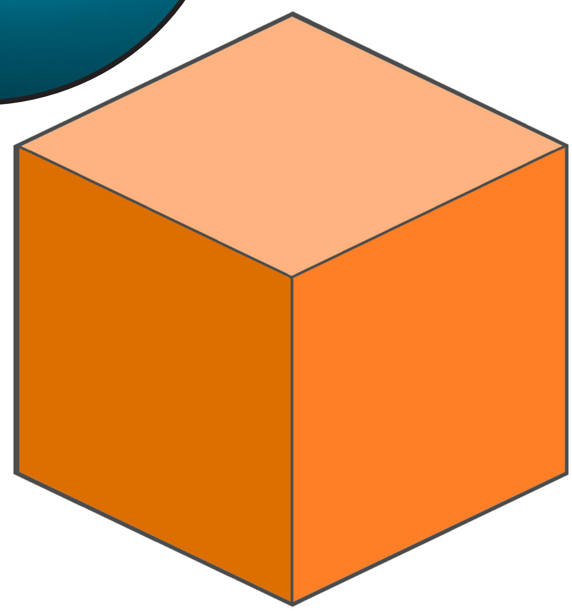
Nested Sphere and Cubes

by Gary S. Flom, MSPE







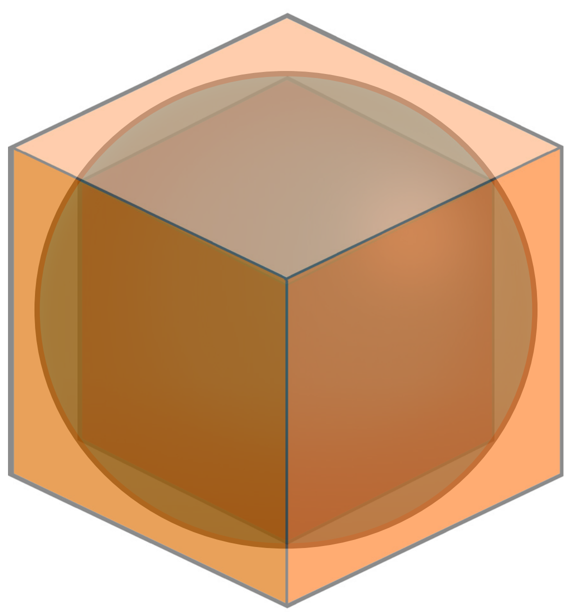
This puzzle can be done completely in your head.

Think of a cube that has 1 unit length on each side.

Now imagine it is nested within a sphere such that all eight corners of the cube intersect with the sphere.

Next, imagine that the sphere is nested within another cube such that the sphere intersects with the outer cube at six points (the middle of each face of the outer cube).

What is the volume of the outer cube?



(Answer can be found in the *Solutions* section at the back of this issue.) **Ω**

Nested Sphere and Cubes: Solution

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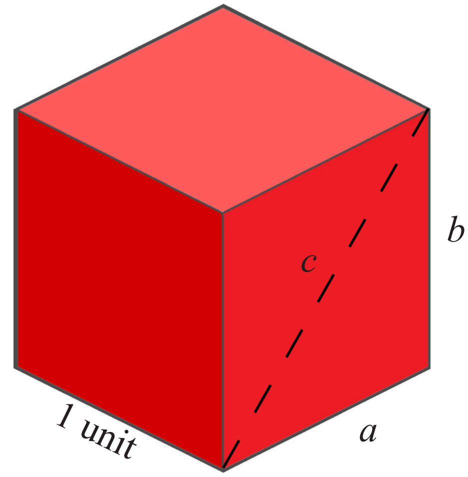
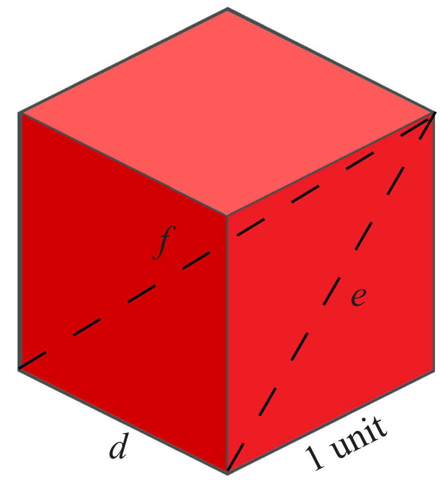
Considering the red cube (the inner cube), draw a diagonal line connecting opposite corners on one face of the cube. The diagonal line creates a right triangle with a length of 1 and a height of 1. Using the Pythagorean Theorem ($a^2 + b^2 = c^2$), we can calculate the length of the diagonal (the hypotenuse). We find $1^2 + 1^2 = c^2$, so $c = \sqrt{2}$.



Next, we need to compute the length of a main diagonal of the inside of the cube. A main diagonal connects opposite corners of the cube. We can find another right triangle using one side of a face (with a length of 1), the diagonal of the face that we just calculated ($\sqrt{2}$), and the main diagonal (the hypotenuse of our new right triangle, side *f*). Using the Pythagorean Theorem, $1^2 + (\sqrt{2})^2 = f^2$, so the length of the main diagonal is $\sqrt{3}$.



Using visualization, we can see that the length of a main diagonal of the red cube ($\sqrt{3}$), is also equal to the diameter of the blue sphere. And by rotating that sphere around, you will notice that the sphere's diameter ($\sqrt{3}$) is also equal to the length of a side of the orange cube (the outer cube).

Because the volume of a cube is length x width x height, we find the volume of the orange cube to be $\sqrt{3} \times \sqrt{3} \times \sqrt{3}$, which is $3\sqrt{3}$. **Ω**