Clock Symmetry

by Gary S. Flom, MSPE

Many clock faces shown in advertising are set to show 10:10 as the time. I have often wondered why, and I have thought of a few ideas. For example, one possible reason is that perhaps it elicits happiness by simulating a smile. Another reason may be that it is symmetric, which many find visually pleasing. Or maybe the clock shows 10:10 so that the hands frame in the watch company's name, as many company names are displayed just below the number 12.

Do any of you have any other ideas about why clocks are often shown with the time 10:10 in advertising materials?

Setting a clock to display 10:10 as the time is close to symmetric, but not quite, as the hour hand will be slightly past the 10.

In the range between 10:00 and 10:15, at what time must a clock be, rounded to the nearest second, so that the hour and minute hands are symmetrical about the vertical?

(Answer can be found in the *Solutions* section at the back of this issue.) Ω

Clock Symmetry: Solution

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We have already been told that the answer will be between 10:00 and 10:15, so we will start with a clock showing a time of exactly 10:00. We want to know what the time will be, rounded to the nearest second, when the hour hand and the minute hand are symmetric around the number 12. Thus, we want to know the time when the distance the minute hand is *past* the number 12 will equal the distance that the hour hand is *before* the number 12.

Let x equal the distance (in number of minutes) that the minute hand travels.

We know that the hour hand moves at 1/12th the speed of the minute hand. So, when the minute hand moves a distance of x, the hour hand moves a distance of 1/12x.

Because we are starting with the time of 10:00, the minute hand is pointing exactly to the number 12 on the clockface, which is at the mark of 0 minutes. The hour hand is pointing exactly to the number 10 on the clockface; this is the mark of 50 minutes past the number 12, which is also 10 minutes *before* the number 12.

So, at any point in time after 10:00 and before 10:15, the distance that the minute hand has traveled past the number 12 will be x. The distance that the hour hand has traveled past the number 10 in that same period of time will be 1/12x. However, the distance *between* the hour hand and the number 12 will be represented as 10 – 1/12x, because the hour hand began at the mark that was 10 minutes before the number 12.

We want to know when these two distances will be the same, so we set these two values equal to each other and solve for x.

$x = 10 - 1/12x$

Adding 1/12x to both sides gives us $13/12x = 10$.

Dividing both sides by 13/12 gives us $x = 9.23$.

Converting this answer to minutes and seconds gives us x = 9 minutes 13.8 seconds, and rounding to the nearest second is 9 minutes 14 seconds.

Thus, at the time 10:09:14, the minute and hour hands are symmetric about the vertical. Ω