

Necessitas Mathematicae

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Through and through the world is infested with quantity: to talk sense is to talk quantities. It is no use saying the nation is large—how large? It is no use saying that radium is scarce—how scarce? You cannot evade quantity. You may fly to poetry and music, and quantity and number will face you in your rhythms and your octaves.

—Alfred North Whitehead (1861-1947)

Since the question has rather casually been raised during one of our ISPE discussions as to what the importance of mathematics to society might be, this essay will endeavor to provide a tentative answer, however reluctantly. As this writer is intimately concerned with mathematics, the reluctance is due to possible mockery of praising one's own horse. Also, apart from being fraught with almost hopelessly vague potential biases, the main difficulty stems from the word "importance."

Whichever way the word "importance" is approached, an inordinate amount of subjectivism inevitably hinders its interpretation. Many more-or-less overlapping concepts compete for well-nigh justifiable usage, such as "significance" and the like. Precious few individuals may agree to the exact meaning of them, let alone their personal weight.

Perhaps an intuitive grasp may be gained by imagining a spectrum on which the concept might range from relatively unimportant to extremely important, with whatever lack of consensus. Something is important when it just cannot be ignored. Most people would agree that when something has attained indispensable importance, it may be somewhat more profitably termed a *necessity*.

That *mathematics is a necessity* is, however, less immediately obvious. To be sure, on an intuitive level, if buttonholed, most people

may agree that mathematics is a serious subject and, therefore, presumably, an important one. Regrettably, this is where the monster of subjectivism bursts in, seizes, and mercilessly devours its hapless victims—leaving but a confusing mess of emaciated bones of contention. Although mathematics is definitely one of the cornerstones of our civilization, this fact is hardly recognized outside circles of erudition where general ignorance and suspicion and confusion still claim more casualties than all disasters ever recorded in our turbulent history. Mathematics is still the most misunderstood of all human fields of inquiry and, somewhat similar to black magic, perhaps the most dreaded. Even if its paramountcy is recognized, it is still feared by all but its most intimate lovers and masters. Just like dentistry—we all who *know* its vital necessity cannot help but hate its torture sessions.

Part of the dreadful misunderstanding stems from a wholly inadequate appreciation of what mathematics really is. Oh, to be sure, mathematics has something to do with *numbers*—even preschoolers know *that* much. Most laymen might even venture to guess that it is the *science of numbers*; but beyond that, they stammer and dodge. Of course, there is no shortage of definitions; as a matter of fact, there is a bewildering plethora of them—none terribly helpful as far as laymen are concerned. The confusion is further compounded by such aspects of mathematics as arithmetic and number theory, planimetry and stereometry, abstract algebra, trigonometry, analytic-projective-non-Euclidean geometry, differential and integral calculus, functional analysis, numerical analysis, vector-tensor calculus, differential geometry and manifolds, matrix-algebra analysis, probability and statistics, multivariate regression-correlation, abstract algebraic-combinatorial topology, and so on, *ad nauseam*. Indeed, to the uninitiated, it does seem like a frightful labyrinth or perhaps nightmarish jungle, which gives rise to those classic approach-avoidance conflicts, which are better known as "math anxiety."

Rare is the individual who has not experienced *math anxiety* in one form or another. Yet this tumor of fear need not become malignant. For most people, it may easily be excised in its benign stage by more individualized education, fascinating teachers, and a better understanding of what mathematics is all about.

A reasonable albeit formal definition is given, for example, by the *Reader's Digest Great Encyclopedic Dictionary* (1975) which explains mathematics as, "The study of quantity, form, arrangement, and magnitude; especially the methods and processes for disclosing, by rigorous concepts and self-consistent symbols, the properties and relations of quantities and magnitudes, whether in the abstract (pure mathematics) or in their practical connections (applied mathematics)."¹ This definition is satisfactorily self-explanatory for all those who can comprehend its concepts and logic. If not, instead of getting discouraged and chucking this aside, read on.

In all fairness to the less sophisticated, and in order to illustrate the necessity of mathematics in however sweepingly lay terms, imagine a *world without mathematics!*

The most striking features of such a world would be general and utter confusion and ubiquitous unrest—i.e., conditions bordering on anarchy and perhaps even chaos, at least as far as its human inhabitants would be concerned. A nonmathematical world would be incomprehensible, unstable, and very unpredictable. Most concepts would be devoid of meaning in any tangible fashion, making even rudimentary consensus impossible—to the point that language would be reduced to gibberish, leaving only signs and grunts as a means of bare communication. Of course, such a terrorizing reign of confusion would mean complete social-structure breakdown, reducing its individuals to a level of helplessness just short of infantile. Social instability would give rise to every nightmarishly evil characteristic of primitive and brutal communities before the dawn of civilized humanity. Such a world would be simply *uncontrollable*, hence *unpredictable*. Whether in

the form of a gigantic madhouse or anything else, such a world would mean the gradual extinction of the human intellect, as we now understand it, as surely as the sun must reach zenith—or, rather, *nadir*, in this sadly depressing case. Who could progress in such a chaotic world, let alone survive a normal lifespan?

Why is all that? Because we live in a *world of quantities*, without which we could hardly make truth-valued statements and judgments. Our logic is validated, every moment of our waking life, by sense perceptions intimately linked to quantities. Our physiological reactions, on the adequacy of which our very survival may depend, are largely determined by environmental information, *which we need to be true, even factually true*. We would rapidly go insane in a world devoid of truth, consistency, and logic. We cannot help it—this is our nature. The mind feeds on truth just as the body feeds on nutrients. In short, we need a truthful world of systematic quantification because our inherent cognitive *modus operandi* depends on it, which, in turn, is an integral functional part of our overall *modus vivendi*.

In a world without mathematics, there would be no systematic quantification. That alone would create a hopelessly confusing state of affairs. After all, whether we like it or not, the physical universe is definitely quantitative. From the smallest visually perceivable objects to the immense celestial bodies, things have dimensions; and such dimensions are quantitative, therefore *naturally* quantifiable. Human sense perception cannot help but appreciate that two rocks are more than one—because such perception is adaptive and has, therefore, definite survival advantages. However, such adaptive perception has to be somehow quantified in order to become meaningfully communicable—which has always been the main business of mathematics, especially that of "natural" mathematics. It may even be said that all modern mathematics represents systematic quantification, in whole or in part, besides whatever else mathematics has become and continues to evolve into in a dynamic fashion characteristic of systematic inquiry.

Indeed, mathematics has become much more than systematic quantification of natural reality. We have come a long way from counting sheep. Modern mathematicians, especially during the past hundred fifty years or so, have even gone so far as to proclaim the independence of modern mathematics from the physical world. Although classical mathematics was believed to have the same kind of foundations as the natural sciences, modern mathematics has renounced its “naturalness” in favor of pursuing its own intrinsic interests. There has been an amazing proliferation of such “new mathematics” as non-Euclidean geometry, set theory, transfinite numbers, infinite-dimensional spaces, abstract algebraic topology, etc., etc. Today, considerably less than 25% of mathematics is applicable to other sciences, and maybe 15-20% is applicable to physical reality—i.e., has physical meaningfulness. To be sure, classical mathematics and classical physics are both very much alive and well and thriving, yet quite behind the aggressive modern developments in both the pure and the applied fields, especially in abstract reasoning.

In a world without mathematics, abstract reasoning would be almost impossible, and ugly disharmony would make art meaningless. Inasmuch as *mathematics is science, philosophy, and art*, in a world without mathematics, all coherence would be gone; cognitive breakdown would follow. Then, of course, neither science nor philosophy nor art could survive, let alone flourish. Now, it would be an ugly exaggeration to make mathematics appear as *the* foundation of science, philosophy, and art—and a patently false one at that. However, if it were suddenly to disappear, then science, philosophy, and art would be rendered almost meaningless thereby. In short, mathematics is the *glue* holding together the scientific-philosophical structures that constitute the *cradle of the mind*—while the cradle is cushioned by art and its beauty, harmony, and esthetic pleasure.

Mathematics itself, to be sure, is a science, a philosophy, and an art; however, it may underlie and hold together the entire intellectual cradle. It is a science because it uses the self-corrective hypothetico-deductive method of

systematic inquiry with its rigorous criteria of reliability, validity, and internal consistency. It is a philosophy by providing quantitative and objective worldviews and by using philosophical methodology to *guarantee* its internal consistency and logic as safeguards against (self) contradiction. Finally, it is an art, as its theorems and proofs and refutations hinge also upon concepts and requirements of beauty, harmony, and elegant simplicity. It even has “schools of thought,” as does art in general, such as classicists, functionalists, intuitionists, abstractionists, modernists, etc., especially in those aspects of mathematics which are claimed to be independent of (physical) reality.

However, its sophistication and esoterics notwithstanding, mathematics is very much related to pragmatic reality, especially as far as laymen are concerned—however fearful, recalcitrant, or ignorant they may be. For it is they who would be the hardest hit in a world devoid of mathematics, precisely because of their fears, ignorance, superstitions, etc., making their lives difficult enough even in a world blessed with mathematics, let alone in a world cursed with the paralyzing absence of it. Paradoxically enough, it is perhaps a somewhat cynical irony that those who need mathematics the most are the last to realize it. As the above sketch tried to illustrate, mathematics is a necessity. For its “anti-champions,” it is more than a necessity—it is *vital*.

“*Navigare necesse est*,” asserted the noble Romans in order to indicate, by means of this simple maxim, the *vital* importance of maritime hegemony for their proud empire, and quite justifiably so—for without supremacy of the Mediterranean, the empire would soon have collapsed. In today’s world, and in light of the foregoing discussion, the maxim might be reformulated as “*computare necesse est*.” While mathematics may be an *uneven* necessity for various societies, groups, and individuals, it is certainly perennial and important enough to be termed a *necessity*; ubiquitous and unequivocal, pleasing or maddening, love it or hate it, *necessitas mathematicae*. Ω